



AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION
JR/INTER SCHOOL DIVISION
SOLUTIONS, CONTEST #6 2011-2012

March 8, 2012

Adviser: Score the contest using the answers below or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

1. Let w = width, $\frac{\text{width}}{\text{length}} = \frac{1}{1.9} = \frac{w}{26}$; $w = \frac{26}{1.9}$ There are 13 stripes, so 1 stripe is $\frac{1}{13}$ of the w or $\frac{1}{13} \left(\frac{26}{1.9} \right) \approx 1.05$ to the nearest hundredth. ANSWER: ≈ 1.05	1. ≈ 1.05																				
2. She needs a minimum of 365×3 or 1065 outfits. Using the counting principle, complete this chart: <table><tr><th>pants</th><th>blouses</th><th>scarves</th><th>outfits</th></tr><tr><td>1</td><td>3</td><td>6</td><td>18</td></tr><tr><td>2</td><td>6</td><td>12</td><td>144</td></tr><tr><td>3</td><td>9</td><td>18</td><td>486</td></tr><tr><td>4</td><td>12*</td><td>24</td><td>1152</td></tr></table> ANSWER: 12 or 12 blouses	pants	blouses	scarves	outfits	1	3	6	18	2	6	12	144	3	9	18	486	4	12*	24	1152	2. 12 or 12 blouses
pants	blouses	scarves	outfits																		
1	3	6	18																		
2	6	12	144																		
3	9	18	486																		
4	12*	24	1152																		
3. Let n represent the number of each type of coin. $0.5n + .1n + .25n = 2.00$ 5 quarters = 1.25 $.4n = 2.00$ 5 dimes = .50 $n = 5$ 5 nickels = .25 \$2.00 ANSWER: 5 quarters, 5 dimes, 5 nickels	3. 5 quarters, 5 dimes, 5 nickels																				
4. Value must be less than \$2500 after year #1 - $5000 - 10\%$ of 5000 = 4500 year #2 - $4500 - 10\%$ of 4500 = 4050 year #3 - $4050 - 10\%$ of 4050 = 3645 year #4 - $3645 - 10\%$ of 3645 = 3280.50 year #5 - $3280.50 - 10\%$ of 3280.50 = 2952.45 year #6 - $2952.45 - 10\%$ of 2952.45 = 2657.20 year #7 - $2657.20 - 10\%$ of 2657.20 = 2391.48 ANSWER: 7 years	4. 7 years																				
5. Let n = the numerator, $n + 3$ = the denominator of the original number. $\frac{3n}{(n+3)+7} = \frac{3}{2}$ $6n = 3n + 30$ $\frac{3n}{n+10} = \frac{3}{2}$ $3n = 30$ The original fraction is $\frac{10}{13}$ ANSWER: $\frac{10}{13}$	5. $\frac{10}{13}$																				
6. When the front end of the train enters the tunnel, the whole length of the train is still outside the tunnel. After the train travels one kilometer, the front end leaves the tunnel, but the whole length of the train is inside the tunnel. After it travels a second kilometer, the rear end of the train leaves the tunnel. The time it takes is the time it takes to travel 2 kilometers. At a speed of 15 km/hr, it takes 4 minutes to travel 1 km or 8 minutes to travel 2 km. ANSWER: 8 minutes	6. 8 minutes																				
7. Sketch each possible figure: (not drawn to scale) ANSWER: (5, 2), (-3, 0), (1, 8) (ordered pairs required)	7. (5, 2), (-3, 0), (1, 8) (ordered pairs required)																				



AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION
JR/INTER SCHOOL DIVISION
SOLUTIONS, CONTEST #5 2011-2012

February 9, 2012

Adviser: Score the contest using the answers below or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

ANSWERS

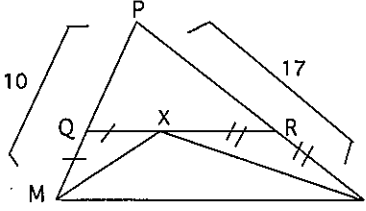
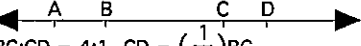
<p>1. Let s represent the number of students.</p> $\frac{1}{2}s + \frac{1}{3}s + \frac{1}{7}s + 20 = s \quad \frac{41}{42}s + 20 = s \quad 840 = s$ <p>ANSWER: 840 students</p>	<p>1. 840 or 840 students</p>
<p>2. Let w represent the length of shorter side. Let l represent the length of the longer side.</p> $2w = l + 5 \quad \frac{1}{3}(w + 9) = \frac{1}{5}l + 3$ $2w - 5 = l \quad \frac{w + 3}{3} = \frac{l}{5} + 3$ <p>by substituting, $\frac{w}{3} = \frac{2w - 5}{5}$</p> $5w = 6w - 15$ $15 = w$ $25 = l$ <p>ANSWER: 15 inches - shorter side, 25 inches - longer side</p>	<p>2. 15 inches - shorter side, 25 inches - longer side</p>
<p>3. Each hour the clocks get 2 minutes apart. In 30 hours they will be one hour apart. When they are actually 12 hours apart, they will look like they are again showing identical times. Therefore, 12 x 30 or 360 hours are required before they again show the identical times. ANSWER: 360 hours</p>	<p>3. 360 or 360 hours</p>
<p>4. Find the prime factors of both 287 and 492. $287 = 7 \times 41$, $492 = 2^2 \times 3 \times 41$. Since 41 is the greatest common factor, this must be the number of spaceships Vulcan has. ANSWER: 41 spaceships</p>	<p>4. 41 or 41 spaceships</p>
<p>5. The first ten prime numbers are : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 Their sum is 129. $129 \div 10 = 12.9$ ANSWER: 12.9</p>	<p>5. 12.9</p>
<p>6. The answer is NOT 100%, although $30\% + 45\% + 25\% = 100\%$</p> $100\% - (100\% - 30\%)(100\% - 45\%)(100\% - 25\%) =$ $100\% - (70\%)(55\%)(75\%) =$ $100\% - 28.875\% = 71.125\%$ <p>ANSWER: 71.125%</p>	<p>6. 71.125%</p>
<p>7. There was \$20 + \$25 + \$35 or \$80 used to purchase lottery tickets.</p> <p>Bill's share was $\frac{25}{80}$ or $\frac{5}{16}$ of 4.8 (million) = 1.5 million.</p> <p>Bill's fair share was \$1.5 million. ANSWER: \$1.5 million</p>	<p>7. \$1.5 million</p>



AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION
JR/INTER SCHOOL DIVISION
SOLUTIONS, CONTEST #4 2011-2012

January 12, 2012

Adviser: Score the contest using the answers below or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

<p>1. To find the two points at equal intervals, first find the difference between $1\frac{7}{15}$ and $\frac{2}{3}$. Then divide this difference by 3.</p> $1\frac{7}{15} - \frac{2}{3} = \frac{12}{15}, \frac{12}{15} \div 3 = \frac{4}{15}$ <p>Add $\frac{4}{15}$ to $\frac{2}{3}$ to get $\frac{14}{15}$. Then add $\frac{4}{15}$ again to get the second point, $1\frac{1}{5}$.</p> <p>These are the 2 points that create equal intervals. ANSWER: $\frac{14}{15}, 1\frac{1}{5}$</p>	<p>1. $\frac{14}{15}, 1\frac{1}{5}$</p>
<p>2. Let P represent the dollar value of Pennypacker's estate.</p> $\frac{P}{2} + 40000 + \frac{1}{2} \left[P - \left(\frac{P}{2} + 40000 \right) \right] + \frac{1}{2} \left[\frac{1}{2} \left(P - \left[\frac{P}{2} + 40000 \right] \right) \right] + 6000 = P$ $\frac{P}{2} + 46000 + \frac{1}{2} \left(\frac{P}{2} - 40000 \right) + \frac{1}{4} \left(\frac{P}{2} - 40000 \right) = P$ $\frac{P}{2} + 46000 + \frac{P}{4} - 20000 + \frac{P}{8} - 10000 = P$ $\frac{7}{8} P + 16000 = P$ $16000 = \frac{P}{8}$ $128000 = P$ <p>ANSWER: 128000</p>	<p>2. 128000</p>
<p>3.</p>  <p>Given: PM = 10 Let a represent QM, PN = 17 Therefore QX = a QM = QR Let b represent RN, XR = RN Therefore XR = b (Not drawn to scale)</p> <p>Perimeter = PQ + PR + QR = (10 - a) + (17 - b) + (a + b) = 10 - a + 17 - b + a + b = 27</p> <p>ANSWER: 27</p>	<p>3. 27</p>
<p>4. (-7) (-5) = 35 (-5) (-1) = 5 (-7) (-1) = 7 (-5) (1) = -5 (-7) (1) = -7 (-5) (3) = -15 (-7) (3) = -21 and so on. ANSWER: -21 is the smallest product</p>	<p>4. -21</p>
<p>5. Let p represent the price of the stereo in dollars. p + 5.5% of p = 410.40 p + 0.055p = 410.40 1.055p = 410.40, p ≈ \$389, 5.5% - 4% = 1.5%. Raul could have saved 1.5% of \$389 or ANSWER: \$5.84</p>	<p>5. \$5.84</p>
<p>6. From the clues: 1 = Jack of spades, 2 = Queen of spades, 3 = Queen of hearts.</p> <p>7. Draw the line (not drawn to scale):</p>  <p>Since AB:AC = 1:3, Since BC:CD = 4:1, CD = $(\frac{1}{4})$BC. AB:BC = 1:2 therefore AB = $(\frac{1}{2})$BC AB:CD = $(\frac{1}{2}) : (\frac{1}{4})$, AB:CD = 2:1 ANSWER: AB:CD = 2:1 (this order required)</p>	<p>6. Jack of spades, Queen of spades, Queen of hearts (in any order)</p> <p>7. AB:CD = 2:1 (this order required)</p>

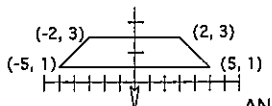


AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION
JR/INTER SCHOOL DIVISION
SOLUTIONS, CONTEST #3 2011-2012

December 8, 2011

Adviser: Score the contest using the answers below or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

ANSWERS

1. Use time = $\frac{\text{distance}}{\text{speed}}$ Car: $t = \frac{28 \text{ miles}}{14 \text{ MPH}}$ $t = 2 \text{ hours}$ Fire: $t = \frac{28 \text{ miles}}{12 \text{ MPH}}$ $t = 2\frac{1}{3} \text{ hours}$ $\frac{1}{3}$ of an hour is 20 minutes ANSWER: 20 minutes	1. 20 minutes
2. $\frac{1}{2 - \frac{1}{2 - \frac{1}{2 - \frac{1}{2}}}} = \frac{1}{2 - \frac{1}{2 - \frac{1}{\frac{3}{2}}}} = \frac{1}{2 - \frac{1}{2 - \frac{2}{3}}} =$ $\frac{1}{2 - \frac{1}{\frac{4}{3}}} = \frac{1}{2 - \frac{3}{4}} = \frac{1}{\frac{5}{4}} = \frac{4}{5}$ ANSWER: $\frac{4}{5}$	2. $\frac{4}{5}$
3. Working backwards, the winners at each step had their points doubled. Therefore, in the preceding play, they had half as much. The loser at each step had to double the points of the other two players. Therefore, at the preceding step the loser had the points he or she now has plus the two amounts given away. The table shows the plays progressing backwards ("W" denotes "winner, "L" denotes "Loser"): End of play third second first #1 40 20 (W) 70 (L) 35 (W) #2 40 20 (W) 10 (W) 65 (L) #3 40 80 (L) 40 (W) 20 (W) ANSWER: 35, 65, 20 in any order	3. 35, 65, 20 in any order
4. Let x = the number of plants at \$0.95. Let y = the number of plants at \$1.15. $x + y = 100$ $0.95x + 1.15y = 100$ $x = 100 - y$ $0.95(100 - y) + 1.15y = 100$ $x = 75$ $95 - 0.95y + 1.15y = 100$ $95 + .2y = 100$ $y = 25$ ANSWER: 75 at \$0.95 each, 25 at \$1.15 each (both answers required)	4. 75 at \$0.95, 25 at \$1.15 (both answers required)
5. Let s represent the number of lambs born $(0.9)(0.55)s = 100$.495s = 100 $s = \frac{100}{.495}$ $s = 202.\overline{02}$ or ANSWER: 203 lambs	5. 203 lambs
6. In each case # stands for "Greatest Common Factor", so $6 \# 6 = 6$. ANSWER: 6.	6. 6
7. Sketch the quadrilateral which is a trapezoid. Let $b_1 = 10$, $b_2 = 4$, $h = 2$. Area _{trap} = $\frac{1}{2}(b_1 + b_2)h$ = $\frac{1}{2}(10 + 4)2$ = 14 square units  ANSWER: 14 square units	7. 14 square units



AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION
JR/INTER SCHOOL DIVISION
SOLUTIONS, CONTEST #2 2011-2012

November 10, 2011

Adviser: Score the contest using the answers below or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

ANSWERS

<p>1. Let A represent the five digit number. $10A + 1 = 3(100000 + A)$ $10A + 1 = 300000 + 3A$ $7A = 299999$ $A = 42857$ The original number was 42857. ANSWER: 42857</p>	1. 42857
<p>2. Since $\frac{2}{3}$ of the pencils are sold, $8 = \frac{1}{3}$ of the pencils. Therefore, I must have started with 24 pencils. $24 \times 15¢ = \\$3.60$ ANSWER: \$3.60</p>	2. \$3.60
<p>3. Since there are 180 degrees in a triangle, the angles in the given ratio can be represented by 2x, 3x and 5x. $2x + 3x + 5x = 180^\circ$ $10x = 180^\circ$ $x = 18^\circ$ $2x = 36^\circ$ The measure of the smallest angle is 36°. ANSWER: 36°</p>	3. 36°
<p>4. The pattern resembles Base 5 $\text{fot} = 5_{10}$ $\text{fotfot} = 10_{10}$ $\text{fotfotfot} = 15_{10}$ $\text{fotfotfotfot} = 20_{10}$ therefore $\text{foefotfotfotfot} = 23_{10}$. ANSWER: foefotfotfotfot</p>	4. foefotfotfotfot
<p>5. Let x represent the number of ounces of water to be added. 30% of 30 is 9 ounces of pure alcohol 25% of $30 + x = 9$ $.25(30 + x) = 9$ $x = 6$ Therefore 6 ounces of water must be added. ANSWER: 6 ounces</p>	5. 6 or 6 ounces
<p>6. $\\$14.90 \div \\2.50 is 5 boxes at 12 each = 60 with \$2.40 remainder. $\\$2.40 \div \\1.25 is 1 packet at 3 each = 3 with \$1.15 remainder. $\\$1.15 \div \\0.50 is 2 single cards = 2 with \$0.15 remainder. There will be $60 + 3 + 2$ or 65 cards bought with \$0.15 left over. ANSWER: 65 or 65 cards</p>	6. 65 or 65 cards
<p>7. Let x represent the number of bills. If x is divided by 5, leaving a remainder of 4, then x must have a units digit of 4 or 9. Since x divided by 8 leaves a remainder of 7, then $x + 1$ must be divisible by 8. Since x divided by 6 leaves a remainder of 3, x must be divisible by 3. The only number less than 100 that meets these conditions is 39. Therefore, $x = 39$ and when x is divided by 7, there are 4 bills left over. ANSWER: 4 or 4 bills left over</p>	7. 4 or 4 bills left over



October 13, 2011

**AMERICAN SCHOLASTIC MATHEMATICS ASSOCIATION
JR/INTER SCHOOL DIVISION
SOLUTIONS, CONTEST #1 2011-2012**

Adviser: Score the contest using the answers below or equivalents. Send the scorecard within 5 days. Make up any missed contests as soon as possible. Include cumulative scores for all students.

ANSWERS

<p>1. Let s represent the number of students taking science. Let h represent the number of students taking history. 10% of $s = 8$ 16% of $h = 8$ $s = 80$ $h = 50$ $80 + 50 = 130$ but 8 are taking both, therefore $130 - 8 = 122$ are taking at least one of the courses. ANSWER: 122</p>	1. 122
<p>2. Lori = 1.46 m Mark = $1.46 + 0.38 = 1.84$ m. Ned = $1.84 - 0.25 = 1.59$ m. therefore, Keith = $1.59 + 0.16 = 1.75$ m. ANSWER: Keith is 1.75 m.</p>	2. Keith is 1.75 m.
<p>3. Game #1 \$1 Yancy must win the <u>eight</u> game (for \$128) since Game #2 \$2 the sum of the first seven games is \$127. A Game #3 \$4 transfer of \$15 will put Yancy ahead of Zeke by Game #4 \$8 \$31. This can only be accomplished if Yancy wins Game #5 \$16 Games #1-4 <u>also</u>. Game #6 \$32 Game #7 \$64 Game #8 \$128 ANSWER: 1st, 2nd, 3rd, 4th and 8th games</p>	3. 1st, 2nd, 3rd, 4th and 8th games
<p>4. Since the chickens were \$9 each, the total 18C43 must be divisible by 9. For a number to be divisible by 9, the sum of its digits must be divisible by 9. C must be 2. To find the number of chickens bought, divide 18243 by 9. $18243 \div 9 = 2027$ ANSWER: C = 2; 2027 chickens</p>	4. C = 2; 2027 chickens (both answers required)
<p>5. Let m represent the number of white mice and p represent the number of parakeets. $m + p = 15$ $4m + 2p = 50$ $p = 15 - m \rightarrow 4m + 2(15 - m) = 50$ $2m = 20; m = 10$ $p = 5$ ANSWER: 5 parakeets</p>	5. 5
<p>6. Let p represent the original number of people in the room. $\frac{1}{2}p + \frac{1}{3}(\frac{1}{2}p) + 12 = p$ $12 = \frac{1}{3}p$ $36 = p$ There were 36 people originally in the room. ANSWER: 36</p>	6. 36
<p>7. Since the length of the diagonal of the rectangle, $BD = 9.5$, and $BD = CE$, $CE = 9.5$. This is also a radius of circle C so the diameter = 2×9.5 or 19. ANSWER: 19 or 19 cm.</p>	7. 19 or 19 cm.